Design of Structural Steel Members Subject to Combined Loading — The Latest

By Abbas Aminmansour, Ph.D.

Moment connections

The 13th edition of the *Steel Construction Manual* published by the American Institute of Steel Construction (AISC) contains a method and aids for design of structural steel members subject to combined loading. The formulas and tables included in Part Six of the manual are based on Chapter H of the 2005 AISC Specification and address design using both Allowable Strength Design (ASD) and Load and Resistance Factor Design (LRFD) methods. While these formulas and tables are easy to use, a better understanding of the concepts behind the method and aids will lead to faster and more efficient design.

The method introduced in Part Six of the manual utilizes certain coefficients in modified versions of AISC Specification Equations H1-1a and H1-1b to check compliance with the Specification. Those modified equations are repeated below for members subject to combined compression and bending.

If, $P_r / P_c \geq 0.2$ (or $pP_r \geq 0.2$)

$$pP_r + b_x M_{rx} + b_y M_{ry} \leq 1.0$$

(*Equation H1-1a, modified*)

If, $P_r / P_c < 0.2$ (or $pP_r < 0.2$)

$$0.5pP_r + (9/8)(b_x M_{rx} + b_y M_{ry}) \leq 1.0$$

(*Equation H1-1b, modified*)

In the above interaction equations, P_r, M_{rx} and M_{ry} are the Required Strengths in compression, bending about the x-axis and bending about the y-axis, respectively. Variables p, b_x and b_y are the corresponding coefficients that may be obtained from Table 6-1 of the AISC Manual. *Figure 1* shows a sample portion of Table 6-1 in the AISC Manual.

As shown in *Figure 2*, coefficients used in this method are measures of member Available Strength for the corresponding type of applied load. For instance, the coefficient p, which is to be multiplied by the Required Strength in compression is the reciprocal of the member Available Strength in compression. Similarly, the coefficient b_x multiplied by the Required Strength for bending about the x-axis is 8/9 times the reciprocal of the member Available Strength about the x-axis.

As illustrated in *Figure 2*, variable p depends on the member effective length (KL) for column action while b_x depends on the unbraced length (L_b) for bending about the x-axis. These values are listed in the main body of Table 6-1 in the Manual. Therefore, the designer should go in the table with the larger of $(KL)_y$ and $(KL)_{y\,eq}$ to obtain the appropriate value for p. In order to obtain b_x, the designer goes in the table with the unbraced length (Lb). Unlike p and b_x, the variable b_y does not depend on any length and is therefore a single value for each section and type of steel. Values of b_y are listed at the bottom of the same table.

Coefficients t_y and t_r used for combined tension and bending are also single values each for a particular steel section and steel type. They are listed at the bottom of the Table 6-1 in the AISC Manual. *Figure 3* shows a sample bottom portion of Table 6-1. Caution should be applied when looking up the appropriate coefficient values for the design method being used, ASD or LRFD.

General Observations for Obtaining Coefficients

The following observations should be noted in using tabulated values of coefficient p, b_x, b_y, t_y and t_r.

1. Values of all coefficients listed in Table 6-1 of the AISC Manual are magnified by 1,000 to avoid excessive decimals in the table. Therefore, tabulated values must be multiplied by 10^{-3} before use in the interaction equations.
2. Compact / non-compact section criteria for flexural buckling and bending about the x and y-axes have been taken into account in developing values of coefficients p, b_x and b_y. Sections with non-compact or slender elements are identified via footnotes in the table.
3. Numbers listed in the far left column of Table 6-1 of the AISC Manual represent the critical effective length (KL) for obtaining p-values and unbraced length (L_b) for obtaining b_x. Note that values of KL and L_b may be different for a particular member.

continued on next page

6–70 DESIGN OF MEMBERS SUBJECT TO COMBINED LOADING

Table 6–1 (continued)
Combined Axial and Bending
W Shapes

W14 — F_y = 50 ksi

Shape	W14×											
	90[f]				82				74			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	1.26	0.840	2.33	1.55	1.39	0.924	2.56	1.71	1.53	1.02	2.83	1.88
6	1.30	0.863	2.33	1.55	1.48	0.983	2.56	1.71	1.63	1.09	2.83	1.88
7	1.31	0.872	2.33	1.55	1.51	1.00	2.56	1.71	1.67	1.11	2.83	1.88
8	1.33	0.882	2.33	1.55	1.55	1.03	2.56	1.71	1.71	1.14	2.83	1.88
9	1.34	0.894	2.33	1.55	1.60	1.06	2.57	1.71	1.76	1.17	2.84	1.89
10	1.36	0.907	2.33	1.55	1.65	1.10	2.61	1.74	1.82	1.21	2.89	1.92
11	1.38	0.921	2.33	1.55	1.71	1.14	2.66	1.77	1.89	1.26	2.94	1.96
12	1.41	0.938	2.33	1.55	1.78	1.18	2.70	1.80	1.96	1.31	2.99	1.99

Figure 1: Sample portion (top) of Table 6-1 of the AISC Steel Construction Manual

4. Values of the coefficient p are based on the critical effective length for column action. Look up the p-value based on the larger of $(KL)_y$ and $(KL)_{y\,eq}$. For convenience, values of (r_x/r_y) are given at the bottom of Table 6-1 for converting $(KL)_x$ to $(KL)_{y\,eq}$.
5. Values of the coefficient b_x are based on the laterally unbraced length for bending about the strong axis. Obtain the b_x-value based on L_b.
6. Tabulated values of b_x account for laterally unbraced length L_b versus L_p and L_r for the sections listed.
7. Values of coefficients p and b_x are not listed for situations where either the slenderness ratio (Kl/r) exceeds 200 or the unbraced length (L_b) exceeds L_r.
8. Values of b_y do not depend on any length such as KL or L_b and are therefore a single value for each steel section and type. They are listed at the bottom of the table on the same page.
9. Values of coefficients b_x and b_y depend on the member flexural strength only and are independent of the type of axial load applied (tension or compression). Therefore, tabulated values of b_x and b_y apply equally for combined tension and bending as well as compression and bending.
10. Values of t_y are based on the actual tension yield strength and depend on cross sectional area and material yield stress. The values of coefficient t_y are a single value for a particular steel section and type. They are listed at the bottom of the table on the same page.
11. Values of t_r are based on an assumed lower bound factor to represent tension rupture strength. It depends on the effective net area. Since effective net area varies and depends on specifics of the connections (e.g. area and location of holes as well as shear lag factor), values of t_r are estimated for $A_e=0.75A_g$. This assumption should be confirmed in design practice. The values of coefficient t_r are listed at the bottom of the table on the same page.

Considerations for Design of Members for Combined Loading

Design for Compression and Bending

Design of members subjected to compression and bending, also known as beam-columns, is a trial-and-error process. Initially, the designer selects a trial section and then checks it for compliance with Equations H1-1a or H1-1b of the AISC Specification or their equivalents in terms of coefficients p, b_x and b_y. The process for design of beam-columns may be summarized as follows.

1. Look through Table 6-1 of the AISC Manual and select a trial section with the desired nominal depth. Look up the values for coefficients p, b_x and b_y. Keep in mind considerations offered earlier for design of members for combined loading.
2. Check the section for compliance with the appropriate interaction equation from page 6-4 of the manual.
3. Continue this process until a satisfactory section is obtained.
4. This design process converges very quickly, even if an "inappropriate" trial section is selected initially. The designer need not spend much time trying to identify a "suitable" first trial section.

Second order effects must be considered in design of beam-columns. However, without a specific section, the values of the moment magnification coefficients B_1 and B_2 are unknown. Therefore, the designer may make initial estimates for coefficients B_1 and B_2 in trying different sections. These values will need to be checked exactly for the final selection.

Design for Tension and Bending

The procedure for design for combined tension and bending is very similar to that for beam-columns with the difference that the larger of t_y or t_r is used instead of p in the interaction equation. See page 6-4 of the AISC Manual for appropriate interaction equations. Again, the reader is reminded that values of t_r listed in Table 6-1 of the AISC Manual are estimates based on $A_e=0.75A_g$. For members with $A_e>0.75A_g$, tabulated values of t_r are conservative. For cases when $A_e<0.75A_g$, exact values of t_r must be calculated.

	LRFD	ASD
Axial Compression	$p = \dfrac{1}{\phi_c P_n}$, (kips)$^{-1}$	$p = \dfrac{\Omega_c}{P_n}$, (kips)$^{-1}$
Strong Axis Bending	$b_x = \dfrac{8}{9\phi_b M_{nx}}$, (kip-ft)$^{-1}$	$b_x = \dfrac{8\Omega_b}{9M_{nx}}$, (kip-ft)$^{-1}$
Weak Axis Bending	$b_y = \dfrac{8}{9\phi_b M_{ny}}$, (kip-ft)$^{-1}$	$b_y = \dfrac{8\Omega_b}{9M_{ny}}$, (kip-ft)$^{-1}$
Tension Rupture	$t_r = \dfrac{8}{\phi_t 0.75 F_u A_g}$, (kips)$^{-1}$	$t_r = \dfrac{\Omega_t}{0.75 F_u A_g}$, (kips)$^{-1}$
Tension Yielding	$t_y = \dfrac{1}{\phi_c F_y A_g}$, (kips)$^{-1}$	$t_r = \dfrac{\Omega}{F_y A_g}$, (kips)$^{-1}$

Figure 2: Equations shown on page 6-3 of the AISC Steel Construction Manual

Design for Biaxial Bending

Design of members subject to biaxial bending is a special case of combined axial load and bending, except that in this case the required axial load P_r is zero. Therefore the interaction diagram becomes: $0 + 9/8(b_x M_{rx} + b_y M_{ry}) = 1.0$. The design process then becomes a simplified version of combined tension and bending or combined compression and bending with the exception that there is no need to obtain a value for p, t_y or t_r.

Additional Benefits of the Coefficients

The coefficients listed in Table 6-1 of the AISC Manual were developed for design of members subject to combined compression and bending, tension and bending or biaxial bending. However, they may be used for analysis or design of beams, columns, and tension members as well, when other aids may not be available. As an example, Table 4-1 of the AISC Manual includes nominal sizes of up to W14's for column design. However, one may use coefficient p from Table 6-1 of the manual to analyze or design a column. For instance, a W18×119 of Grade 50 steel with a critical effective length of 15 feet has a p-value of 1.32×10^{-3} (kips)$^{-1}$ in ASD and 0.879×10^{-3} (kips)$^{-1}$ in LRFD. Considering the fact that p is the reciprocal of the Available Strength (*Figure 2*), the Available Strength of this member in axial compression is P_n/Ω_c = 758 kips in ASD and $\Phi_c P_n$ = 1,140 kips in LRFD.

In a similar manner, coefficient b_x may be used to determine the Available Strength of a beam for a given unbraced length using equations illustrated in *Figure 2*. It is noted, however, that values of b_x listed in Table 6-1 of the AISC Manual are based on C_b = 1.0 representing a member subjected to uniform flexure. It is not uncommon for beam-

Other Constants and Properties						
$b_y \times 10^3$ (kip-ft)$^{-1}$ $t_y \times 10^3$ (kips)$^{-1}$ $t_r \times 10^3$ (kips)$^{-1}$	4.90 1.26 1.55	3.26 0.840 1.03	7.95 1.39 1.71	5.29 0.924 1.14	8.80 1.53 1.88	5.85 1.02 1.26
r_x/r_y	1.66		2.44		2.44	

[f] Shape does not meet compact limit for flexure with F_y = 50 ksi.

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Figure 3: Sample portion (bottom) of Table 6-1 of the AISC Steel Construction Manual

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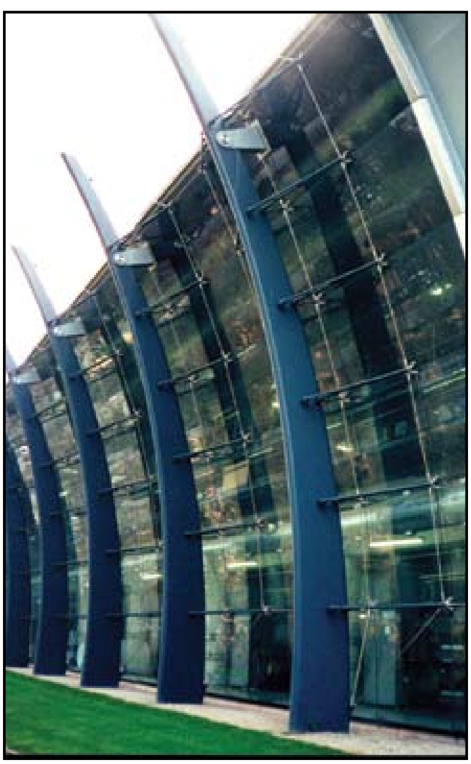
columns encountered in practice to have C_b > 1.0 which increases the flexural strength of the member. The designer is encouraged to utilize the potential additional strength by incorporating C_b when appropriate, keeping in mind that the member nominal strength is limited to M_p.

Consider a W21×48 beam of A992 steel with L_b = 12 feet and C_b=1.0. From Table 6-1, this member has a b_x value of 4.30×10^{-3} (kip-ft)$^{-1}$ in ASD and 2.86×10^{-3} (kip-ft)$^{-1}$ in LRFD. Using the appropriate equations from *Figure 2*, one obtains M_n/Ω_b = 207 kip-ft in ASD and $\Phi_b M_n$ = 311 kip-ft in LRFD for Available Strengths in bending. Note that in this case, L_p = 6.09 ft < L_b = 12 ft < L_r = 16.6 ft, but Table 6-1 already accounts for this condition. Further, footnotes in Table 6-1 (page 6-49) notes that "Shape is slender for compression with F_y = 50 ksi" and "Shape does not meet compact limit for flexure with F_y =50 ksi." However, all that information is already accounted for in developing Table 6-1 as well.▪



Structural steel members –Western Morning News Building, Plymouth, England

W14

Table 6–1 (continued)

Combined Axial and Bending

W Shapes

F_y = 50 ksi

Shape	W14×							
	90[f]				82				74			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
Effective length KL (ft) with respect to least radius of gyration r_y or Unbraced Length L_b (ft) for X-X axis bending	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	1.26	0.840	2.33	1.55	1.39	0.924	2.56	1.71	1.53	1.02	2.83	1.88
6	1.30	0.863	2.33	1.55	1.48	0.983	2.56	1.71	1.63	1.09	2.83	1.88
7	1.31	0.872	2.33	1.55	1.51	1.00	2.56	1.71	1.67	1.11	2.83	1.88
8	1.33	0.882	2.33	1.55	1.55	1.03	2.56	1.71	1.71	1.14	2.83	1.88
9	1.34	0.894	2.33	1.55	1.60	1.06	2.57	1.71	1.76	1.17	2.84	1.89
10	1.36	0.907	2.33	1.55	1.65	1.10	2.61	1.74	1.82	1.21	2.89	1.92
11	1.38	0.921	2.33	1.55	1.71	1.14	2.66	1.77	1.89	1.26	2.94	1.96
12	1.41	0.938	2.33	1.55	1.78	1.18	2.70	1.80	1.96	1.31	2.99	1.99
13	1.44	0.956	2.33	1.55	1.85	1.23	2.75	1.83	2.05	1.36	3.05	2.03
14	1.47	0.976	2.33	1.55	1.94	1.29	2.79	1.86	2.14	1.43	3.10	2.07
15	1.50	0.998	2.33	1.55	2.04	1.36	2.84	1.89	2.25	1.50	3.16	2.10
16	1.54	1.02	2.35	1.57	2.15	1.43	2.89	1.92	2.38	1.58	3.22	2.15
17	1.58	1.05	2.38	1.59	2.28	1.52	2.94	1.96	2.52	1.67	3.29	2.19
18	1.62	1.08	2.42	1.61	2.42	1.61	3.00	1.99	2.67	1.78	3.35	2.23
19	1.67	1.11	2.45	1.63	2.58	1.71	3.05	2.03	2.85	1.89	3.42	2.28
20	1.72	1.14	2.48	1.65	2.75	1.83	3.11	2.07	3.04	2.02	3.50	2.33
22	1.83	1.22	2.55	1.70	3.18	2.12	3.23	2.15	3.51	2.34	3.65	2.43
24	1.97	1.31	2.62	1.74	3.73	2.48	3.37	2.24	4.12	2.74	3.82	2.54
26	2.12	1.41	2.70	1.79	4.38	2.91	3.51	2.34	4.83	3.22	4.00	2.66
28	2.31	1.53	2.78	1.85	5.08	3.38	3.67	2.44	5.61	3.73	4.20	2.80
30	2.52	1.68	2.86	1.91	5.83	3.88	3.84	2.55	6.44	4.28	4.42	2.94
32	2.77	1.85	2.95	1.97	6.63	4.41	4.03	2.68	7.32	4.87	4.73	3.15
34	3.07	2.04	3.05	2.03	7.49	4.98	4.28	2.85	8.27	5.50	5.09	3.38
36	3.43	2.28	3.16	2.10	8.39	5.59	4.57	3.04	9.27	6.17	5.44	3.62
38	3.82	2.54	3.27	2.17	9.35	6.22	4.86	3.24	10.3	6.87	5.80	3.86
40	4.23	2.81	3.39	2.25	10.4	6.90	5.15	3.43	11.4	7.61	6.15	4.09
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$ $t_y \times 10^3$ (kips)$^{-1}$ $t_r \times 10^3$ (kips)$^{-1}$	4.90 1.26 1.55		3.26 0.840 1.03		7.95 1.39 1.71		5.29 0.924 1.14		8.80 1.53 1.88		5.85 1.02 1.26	
r_x/r_y	1.66				2.44				2.44			

[f] Shape does not meet compact limit for flexure with F_y = 50 ksi.