Punching Shear in Thin Foundations

By Dan Mazzei, P.E.

In 2010, Wallace Engineering (Tulsa, OK) was asked to perform the structural engineering for a 137,000 square foot, single story commercial building in Puerto Rico. Load conditions at this site are a combination of the 145 mph winds found in Florida, California level seismic loads, soft site class E soils found in the Carolina coastal areas, and a minimum live load of 40 psf for low slope roofs. The structure was a steel bearing frame, supported laterally by precast shear walls on shallow spread and strip footings, with a polished concrete slab interior floor surface. Additionally, the geotechnical engineer had limited footing bearing depths to a maximum of 18 inches below grade. This maximum restriction was based on the geotechnical engineer's assumption that a certain thickness of engineered fill was required to span over a layer of soft soils below the building's footprint. Understanding the more costly engineering and detailing that would be required to accommodate this limit, Wallace Engineering first attempted to persuade the geotechnical engineer to reconsider his conclusions. Initial attempts were unsuccessful in getting the recommendations changed, and the analysis and design began. Additionally, the owner required that the interior floor be a crack-free polished concrete slab, which limited the top of footing elevation to 6 inches below finished floor and provided only 12 inches of depth for the footing.

The maximum ultimate gravity load was 220 kips. Normally, a spread footing could be made as wide and thick as required to support this level of ultimate load. However, the maximum bearing depth restriction prevented the use of a thicker footing. Instead, the footing was reinforced so that it could support the shear requirements detailed in ACI 318-08 11.11.2.1 and ACI 318-08 11.11.1.2, as well as the tension forces from wind uplift loads illustrated in ASCE 7-05 Figure 6-6. The procedure used to satisfy the design requirements of the footing was as follows:

A. Check base plate to support the ultimate gravity load.
B. Check punching shear capacity of the thin footing per ACI 318 15.5.
C. Determine increased punching shear capacity from raising concrete compressive strength.
D. Use flexural theory to size a rigid base plate that would justify increasing the loaded area on the footing from half the distance between the face of the column and the edge of the base plate, per ACI 318 15.4.2.C, to the full width of the base plate.
E. Design a reinforced shear head, per ACI 318 11.12.3, within the footing to distribute the gravity loads sufficiently throughout the footing to further increase the effective punching shear capacity. Then, verify the flexural capacity of the portion of the footing surrounding the shear head, per ACI 318 15.4, for the ultimate gravity load. (Note: This solution permits the use of a typical steel base plate)

Once the shear head and footing are properly designed for gravity loads, the footing must then be checked for both global uplift and also flexure (again per ACI 318 15.4) due to its size. Lastly, the anchor bolt group at the column must be sized, and the reinforcing ties properly arranged and considered to prevent ACI 318 Appendix D pullout.

Items A-E are explained in more detail below. Also, it should be noted that, to set up the rigid base plate calculation, the base plate bearing calculation is described in detail. This may seem redundant but is useful in fully understanding the concepts used during the later bending check.

A. Check bearing capacity of base-plate to support the ultimate gravity load:
   - $P_{ultimate}$ = 220 kips (after live load reduction per IBC 1607.11.2.1)
   - f'_c = concrete compressive strength = 3500 psi
   - A_1 = width and length of square base plate (*Figure 1, page 56*)
   - A_2 = area of concrete below plate (Wallace Engineering uses this size unless smaller) = (24 in.)(24 in.) = 576 in.^{2}
   - Base plate supports HSS 8x8 column and has (4) anchor bolts with 1½-inch clear from the edge of the base plate to centerline of bolt.
   - Per equation J8-2 of AISC 13th Edition, the limit state of concrete crushing is:

$$\phi P_p = \phi(0.85f'_c A_1)(A_2/A_1)^{1/2} < \phi 1.7 f'_c A_1$$

Where

$0.85f'_c A_1$ = Bearing strength on concrete per ACI 318 10.14.1 when the supporting area is not wider than the base plate.

$(A_2/A_1)^{1/2}$ = Permitted increase when the loaded area of concrete is wider on all sides of base plate because the loaded area is confined by surrounding concrete (again, the total increase must be < 2 per ACI 318)

$1.7f'_c A_1$ = Upper limit so that increase from $(A_2/A_1)^{1/2}$ is less than 2, per ACI 318 (i.e. (2)(.85) = 1.7)

In the absence of code provision, conservatively use ϕ = .6 per AISC Section J8, but could use ϕ = .65 per ACI 318 9.3.2.4. Since A_2 = 576 square inches, we can solve for A_1 and take advantage of the concrete below the base plate being confined by the concrete outside the base plate's footprint (i.e. $(A_2/A_1)^{1/2} < 2$ translates into the upper limit of $(1.7)(.85f'c)(A_1)$). Therefore, A_1 equals the greater of the following:

$$A_1 = (1/576 \text{ in.}^2)[(220 \text{ kips}/((0.6)(.85)(3.5 \text{ ksi}))]^2 = 26.4 \text{ in.}^2$$
$$A_1 = 220 \text{ kips}/[(0.6)(1.7)(3.5 \text{ ksi})] = 61.2 \text{ in.}^2$$

The required base plate width for concrete bearing is simply the square root of A_1, or 7.85 inches. However, since the columns are HSS 8x8s and a 1½-inch clear spacing will be specified between the center-line of the anchor bolt and the edge of the base plate, the base plate size will be increased to 14x14 inches (see *Figure 1*).

Therefore, actual area under the base plate is A_1 = (14 in.)(14 in.) = 196 square inches. The area of concrete support below the base plate is still A_2 = 576 square inches.

continued on next page

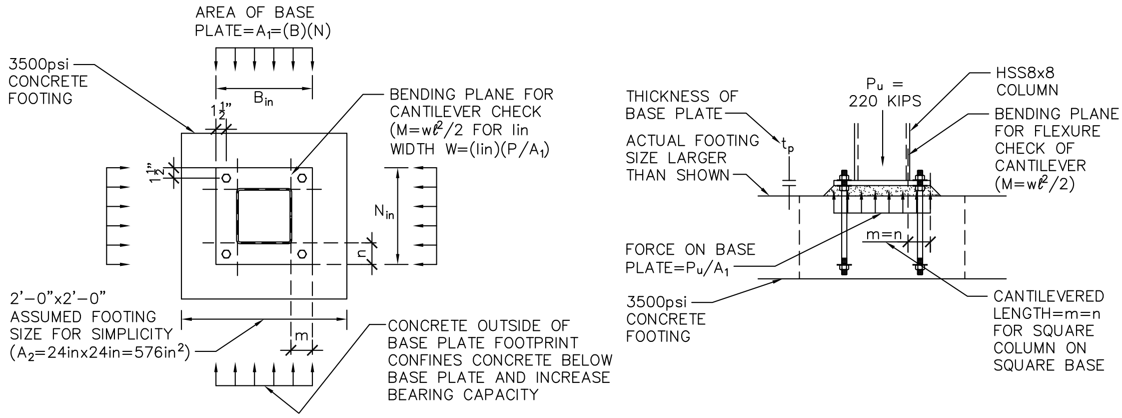

Figure 1: Bearing plate check.

Therefore, the capacity of the selected base plate for concrete crushing is ϕP_p = (.6)(0.85)(3.5 ksi)(196 in.^{2})(576 in.^{2}/196 in.^{2})$^{1/2}$ = 599.8kips > 220 kips ⇐ <u>OK for Concrete Crushing</u>
Since the assumed HSS 8x8 base plate width of 14x14 inches is acceptable with respect to concrete crushing, the bending capacity of the plate must be checked. The base plate width (B) = 14 inches and the base plate length (N) = 14 inches. With the base plate's bending plane being near the face of the column, and considering the portion of the base plate beyond the face of the column as cantilevered out some length beyond that point, the maximum cantilever length is the greater of m, n and λn´:

Per 13th Edition AISC part 14 and *Figure 1*, for a square HSS shape m = n = [N – (0.95)(column width)]/2 = [14 in. – (0.95)(8 in.)]/2 = 3.2 in.

This must be compared to a cantilevered length based on yield line theory, also referred to as λn´. (Note: λn´ will not control when the base plate is this much larger than the supported column, but the check is included for reference)
n´ = [(overall column depth)(column flange width)]$^{1/2}$/4 = [(8 in.)(8 in.)]$^{1/2}$/4 = 2 in.
$\lambda_s = 2(X)^{1/2}/(1+(1-X)^{1/2}) \leq 1$
X = [(4)(overall column depth)(column flange width)/(overall column depth + column flange width)^2]($P_u/\phi P_p$)
λ_s could be conservatively taken as 1, however solving for X and then for λ_s:

X = [(4)(8 in.)(8 in.)/(8 in. + 8 in.)^2] (220 kips/599.8 kips) = 0.37
$\lambda_s = (2)(0.37)^{1/2}/(1+(1-0.37)^{1/2}) = .68$
$\lambda_s n' = (0.68)(2) = 1.36$ in.

For flexure design, the longest cantilever length controls. In this case, that is m = n = 3.2 inches.
Viewing the cantilevered plate as a uniformly loaded 1-inch wide strip, the maximum moment is near the face of the column and the load on the plate is w = (P/A_{eff})(1 in.). Therefore, the maximum moment at the support of a cantilever, $wl^2/2$ can be expressed as (1 in)$(P_u/A_{eff})(l^2)/2$. Since the plastic section modulus, $Z = t_{plate}^2/4$ and the nominal moment, $\phi M_n = \phi M_p = \phi(F_y)(Z)$, the expressions are combined and the equation for t_{plate} is derived as follows:
ϕ = 0.9 for flexure
t_p = (max. cantilevered length) $[(2)(P_u/A_{eff})(1/.9F_y)]^{1/2}$
t_p = 3.2 in[(2)(220 kips/(196 in^2)(1/(.9)(36 ksi)]$^{1/2}$ = .84 in.

Therefore, to satisfy initial bearing requirements, a ⅞-inch thick base plate is required. This same theory is defined in Part 14 of the 13th Edition of AISC and will also be used to size the rigid base plate.

B. As shown in *Figure 2*, with the applicable ACI 318 references, the punching shear capacity is shown to be:
ϕ for shear = 0.75 per ACI 318 9.3.2.3 (Note: the lower ϕ value due to increased dependence on concrete quality for shear strength)
λ_c for normal weight concrete = 1.0
ϕVc = (0.75)(4)(1.0)(3500 psi)$^{1/2}$(19 in.)(4 sides)(8 in.)(1 kip/1000lbs) = 107.9 kips < 220 kips ⇐ <u>No Good For Punching Shear</u>

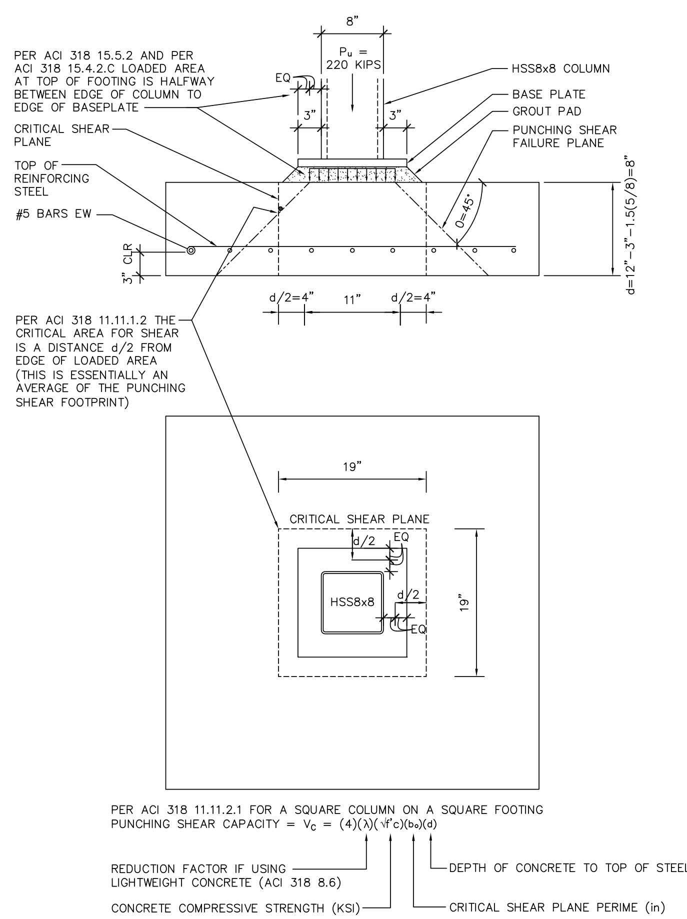

Figure 2: Punching shear.

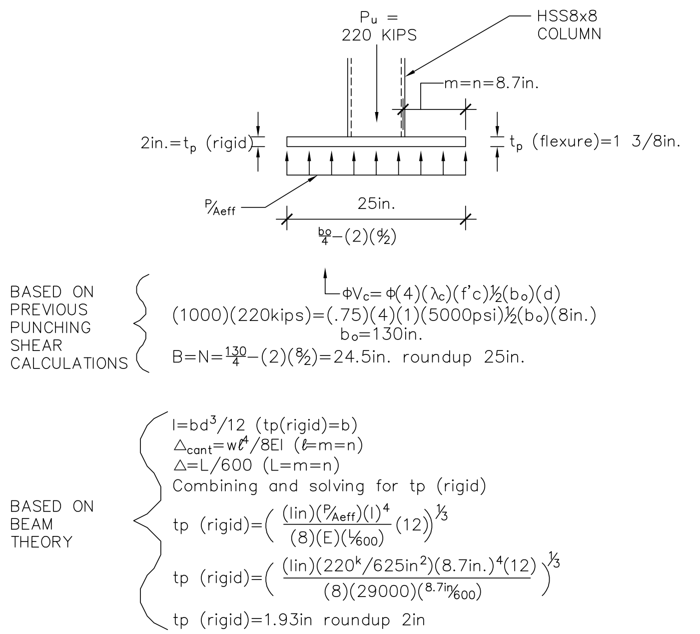

Figure 3: Rigid base plate calculation.

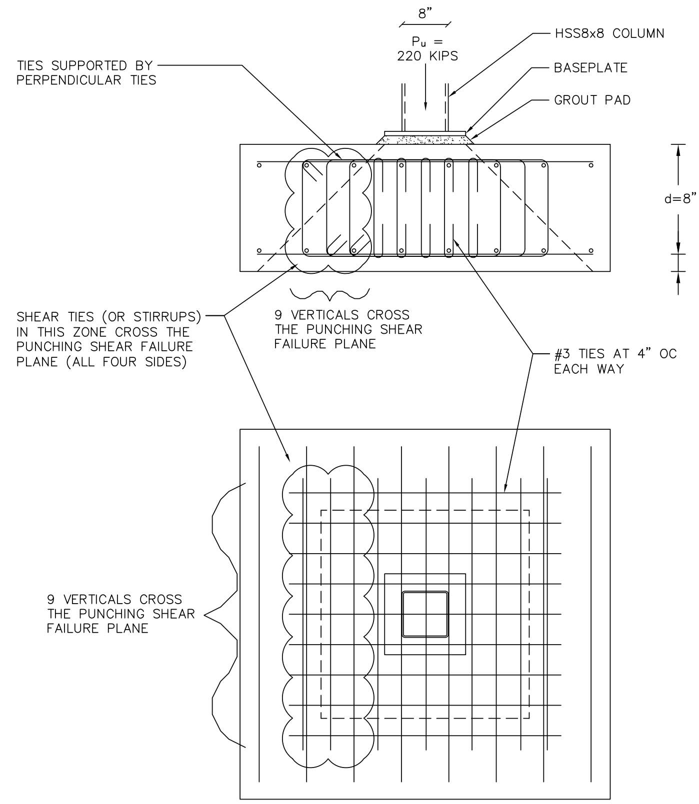

Figure 4: Shear head reinforcement.

Since soft soils were an issue for this site, and since the design bearing pressure was 1500 psf, soil pressure offset was disregarded in the punching shear calculations. With that said, at this point one would normally simply increase the thickness of the footing until the punching shear check was satisfied. If that had been an option on this project, increasing the depth of the footing by 6 inches to a total depth of 18 inches would have provided a punching shear capacity equal to $\phi Vc = 248.5$ kips > 220 kips, flexural capacity would have been verified per ACI 318 15.4 and the footings would be acceptable. A thicker footing in this case is the preferable and more cost effective solution. However, due to the maximum bearing depth constraints, deeper footings were not an option. Therefore, an alternative was necessary.

C. The increased capacity from a higher compressive strength concrete, although minor due to the tensile nature of shear failure, is still worth checking. Using $f'_c = 5000$ psi. along with the procedure shown above provides another 21 kips of capacity:

$\phi Vc = (0.75)(4)(1.0)(5000\text{ psi})^{1/2}(19\text{ in.})(4\text{ sides})(8\text{ in.})(1\text{ kip}/1000\text{ lbs}) = 128.9$ kips << 220 kips ⇐ No Good For Punching Shear

D. To further increase the punching shear capacity, a rigid base plate can be sized by holding the deflection of the previously defined cantilever to L/600. The assumption being that such a rigid plate could distribute its load over its entire foot-print (in lieu of just up to half the distance from its edge to the face of the column) and engage enough of the concrete footing to achieve the necessary critical shear area. Per *Figure 3*, this resulted in a 25-inch x 25-inch x 2-in-thick base plate.

However, with the top of footing only 6 inches below finish floor, the top of the anchor bolts penetrate significantly into the previously mentioned interior of the polished concrete slab. This penetration increased the potential for unattractive cracks in the polished slab. Therefore, to utilize more typically sized column base plates, the decision was made to add a shear head within the footing to distribute the punching shear forces over a wide enough area so that the thin footing could handle the applied ultimate load.

E. Design Shear Head per ACI 318 11.11.3. As noted in the rigid base plate calculations, the required critical shear area length, b_o, is just under 130 inches. Therefore, any shear head installed inside the footing must be able to distribute P_u out to this line. If the shear head is considered a reinforced concrete beam within the footing that extends from where P_u is applied to the edge of the critical shear area, the following results:

- Assume the beam within the footing to be as wide as the critical shear area, then the cross-section of the beam is 32.5 inches x 12 inches
- Assume the beam has #3 shear ties at 4 inches OC each way the full length of the beam, and assume it has (4) #6 bars each way top and bottom for flexural reinforcement.
- Per ACI 318 11.1.1 total shear capacity is $V_n = V_{steel} + V_{concrete}$
- Per ACI 318 11.11.3, shear reinforcement is permitted since the calculated d of 8 inches exceeds both 6 inches and (16)(bar diameter).
- Per ACI 318 11.4.7.2 and *Figure 4*, the shear capacity of steel, $\phi V_s = \phi A_s F_y d/s$

  ϕ = reduction factor per ACI 318 9.3.2.3 = .75 for shear

  A_s= area of shear reinforcement = (0.11 in.^{2})(9 verticals) = 0.99 in.^{2}

  F_y = shear reinforcement yield strength = 60 ksi

  d = depth from top of concrete to top of bottom reinforcing steel reinforcement = 8 inches

  s = spacing of shear reinforcement along shear failure plane = 4 inches

continued on next page

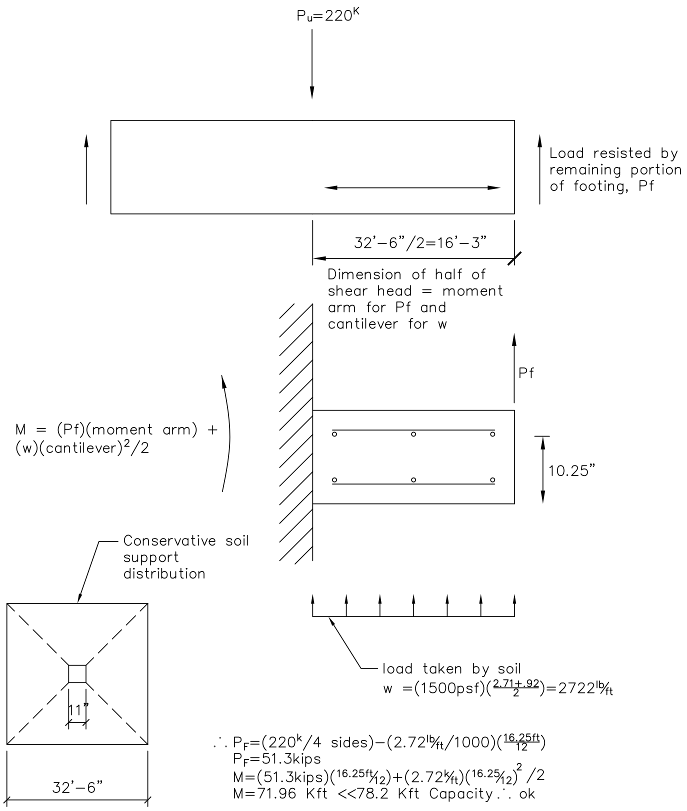

Figure 5: Shear head flexure check.

Per *Figure 4*, the #3 ties are spaced at 4 inches OC each way so that (9) verticals crossed the punching shear failure plane. Their hook lengths were sized per ACI 318 7.1.3.a as (6)(db) = (6)(⅜ in.) = 2.25 inches. Ties were spaced at this interval within the entire shear head so that they would both cross the shear failure plane and also fully engage the bottom steel. Full engagement of the bottom steel is necessary to prevent anchor bolt pullout in uplift, per ACI 318 Appendix D.

$\phi V_s = (0.75)(0.99\text{in.}^2)(8\text{ in.})(60\text{ ksi})/(4\text{ in.}) = 89.1$ kips

- Per ACI 318 11.2.1.1 the shear capacity of the concrete, $\phi V_c = \phi 2(f'c)^{1/2}(b)(d)$

  ϕ = reduction factor per ACI 318 9.3.2.3 = .75 for shear
  f'_c = compressive strength of concrete = 5000 psi
  b = width of concrete beam, in inches
  d = depth from top of concrete to top of bottom reinforcing steel reinforcement = 8 inches

  $\phi V_c = (0.75)(5000)^{1/2}(32.5\text{ in.})(8\text{ in.}) = 27.6$ kips

  Therefore $\phi V_n = \phi V_s + \phi V_c = 89.1$ kips + 27.6 kips = 116.7 kips on one side of the critical shear plane. However, the beam crosses the plane at four locations; therefore, the shear capacity of the shear head = (4)(116.7 kips) = 466 kips >> 220 kips ⇐ <u>OK At Shear Head For Punching Shear</u>

- Per ACI 318 Chapter 10 and *Figure 5*, the flexural capacity of the shear head beam (within the footing) = $\phi Mn = \phi A_s F_y d(1\text{-}.59\rho Fy/f'c)$

  ϕ = reduction factor per ACI 318 9.3.2.1 = .9 for flexure
  b_o = 130 inches and therefore beam cantilever length = (130in/4)/2 = 16.25 inches
  A_s = area of tensile reinforcement = ρbd
  ρ = the balanced steel ratio (between p_{max} and p_{min} per ACI 318 10.3.3 and ACI 10.5.2) = $A_s/bd = (4)(.44\text{ in}^2)/(32.5\text{ in.})(8\text{ in.}) = 0.0067$
  F_y = steel reinforcement yield strength = 60 ksi
  f'c = compressive strength of concrete = 5000 psi
  b = width of concrete beam, in inches
  d = depth from bottom of concrete to bottom reinforcing steel = 10.25 in.

  Therefore, the flexural strength of the beam within the footing resolves to $\phi Mn = \phi\rho F_y bd^2\ [1\text{-}(0.59)(\rho)(F_y)/(f'_c)]$

  Therefore, $\phi Mn = (0.9)(0.0067)(60\text{ ksi})(32.5\text{ in.})(10.25\text{ in.})^2(1/12)[1\text{-}(.59)(0.0067)(60\text{ ksi})/(5\text{ ksi})] = 78.2$ kip-ft

  The required moment at the shear head = 71.2 kip-ft, calculated per *Figure 5*.

  Since the moment capacity of the beam in the shear head = 78.2 kip-ft >> 71.2 kip-ft ⇐ <u>OK At Shear Head For Flexure</u>

  Since the shear head (or beam within the footing) checks out for shear and for bending, the entire footing's flexural capacity must be checked per ACI 318 15.4. This last check requires that reinforcing steel be added or that the shear head size be increased. Once that is accomplished, the footing can support an ultimate load of P_u = 220 kips.

Once the footing is designed for gravity loads (per the above procedure) and soil bearing capacity, the uplift forces must be accounted for. Using wind load pressures from 145 mph winds per ASCE 7-05 Figure 6-6, base plate thickness is determined based on the yield moment of the base plates per 13th Edition AISC, anchor bolt pullout is checked against ultimate uplift forces per ACI 318 Append D and finally, per ACI 318 13.2.1, the extra large footings must be designed with sufficient flexural strength to ensure the entire footing is engaged to resist the maximum applied uplift. Lastly, the footings must be tied together per the requirements in International Building Code (IBC) Chapter 18 for seismic design category D structures built on site class E soils.

In the end Wallace Engineering convinced the owner to pay the geotechnical engineer to perform an additional analysis of the building pad and to verify its ability to support deeper footings. After the new analysis was completed, the geotechnical engineer revised his recommendations to permit a maximum bearing depth of 30 inches. Per the calculations above, this allowed the footing to be made sufficiently deep to pass the bearing, punching shear, flexure, and uplift checks. Therefore, a workable solution was developed to solve the original problem, and the shear heads were not required.▪

Dan Mazzei, P.E. is an Associate at Wallace Engineering, headquartered in Tulsa, Oklahoma. He is a former U.S. Army officer, a member of the American Concrete Institute, and a member of the Oklahoma Structural Engineers Association. Dan can be reached at ***dmazzei@wallacesc.com****.*